
**NEW YORK STATE TEACHER
CERTIFICATION EXAMINATIONS™**

**FIELD 04: MATHEMATICS
TEST FRAMEWORK**

June 2003

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**New York State Teacher
Certification Examinations™**

**FIELD 04: MATHEMATICS
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Subarea

	Selected-Response	Range of Objectives
I.	Mathematical Reasoning and Communication	0001–0003
II.	Algebra	0004–0009
III.	Trigonometry and Calculus	0010–0013
IV.	Measurement and Geometry	0014–0017
V.	Data Analysis, Probability, Statistics, and Discrete Mathematics	0018–0021
	Constructed-Response	
VI.	Algebra: Constructed-Response Assignment	0004–0009

NEW YORK STATE TEACHER CERTIFICATION EXAMINATIONS™

FIELD 04: MATHEMATICS TEST FRAMEWORK

Mathematical Reasoning and Communication
Algebra
Trigonometry and Calculus
Measurement and Geometry
Data Analysis, Probability, Statistics, and Discrete Mathematics
Algebra: Constructed-Response Assignment

The New York State mathematics educator has the knowledge and skills necessary to teach effectively in New York State public schools. The mathematics teacher is adept at utilizing the mathematical systems of algebra, geometry, trigonometry, and calculus and can address and solve problems involving data analysis, probability, statistics, and discrete mathematics. The mathematics teacher is able to reason logically and understands the connections between mathematics and other disciplines. Most importantly, the mathematics teacher is able to communicate mathematically and use language skills to explain mathematical concepts and processes, is able to apply mathematics in real-world settings, and is able to solve problems through the integrated use of multiple mathematical skills and concepts.

SUBAREA I—MATHEMATICAL REASONING AND COMMUNICATION

0001 Understand reasoning processes, including inductive and deductive logic and symbolic logic.

For example:

- analyzing mathematical situations by gathering evidence, making conjectures, formulating counterexamples, and constructing and evaluating arguments
- analyzing the nature and purpose of axiomatic systems (including those of the various geometries)
- analyzing and interpreting the truth value of simple and compound statements (e.g., negations, disjunctions, conditionals) in truth tables and Venn diagrams
- using laws of inference to draw conclusions and to test the validity of conclusions
- applying the principle of mathematical induction to prove theorems

**FIELD 04: MATHEMATICS
TEST FRAMEWORK**

0002 Understand the meaning of mathematical concepts and symbols and how to communicate mathematical ideas in writing.

For example:

- translating among algebraic, graphic, numeric, and written modes of presenting mathematical ideas
- converting between mathematical language, notation, and symbols and standard English language
- deducing the assumptions inherent in a given mathematical statement, expression, or definition
- evaluating the precision or accuracy of a mathematical statement

0003 Understand mathematical modeling and apply multiple mathematical representations to connect mathematical ideas and solve problems.

For example:

- evaluating mathematical models (e.g., graphs, equations, physical and pictorial representations) and analyzing their appropriateness, efficiency, and accuracy in solving a given problem
- analyzing techniques of estimation and identifying situations in which estimation is appropriate
- using estimation to evaluate the reasonableness of a solution to a problem
- representing problem situations using a variety of mathematical models (e.g., algebraic expressions, sequences, diagrams, geometric figures, graphs)
- analyzing the use of software (e.g., simulations, spreadsheets) to model and solve problems

**FIELD 04: MATHEMATICS
TEST FRAMEWORK**

SUBAREA II—ALGEBRA

0004 Understand principles and properties of the set of complex numbers and its subsets.

For example:

- applying principles of number theory (e.g., prime numbers, divisibility) to solve problems
- applying number concepts (e.g., fractions, percents, exponents) to solve problems
- applying knowledge of real numbers to arithmetic and algebraic operations
- justifying the need for the extension of a given number system
- using multiple representations of the complex numbers and their operations (e.g., polar form; algebraic and geometric interpretations of the sum, difference, and product of complex numbers)
- analyzing and applying the properties of vectors, groups, and fields to the complex numbers and its subsets

0005 Understand the principles and properties of patterns and algebraic operations and relations.

For example:

- determining algebraic expressions that best represent patterns among data presented in tables, graphs, and diagrams
- generalizing patterns using explicitly defined and recursively defined functions
- performing and analyzing basic operations on numbers and algebraic expressions
- deriving an algebraic model that best represents a given situation and evaluating the strengths and weaknesses of that model
- applying algebraic concepts of relation and function (e.g., range, domain, inverse) to analyze mathematical relationships
- analyzing the results of transformations (e.g., translations, dilations, reflections, rotations) on the graphs of functions

**FIELD 04: MATHEMATICS
TEST FRAMEWORK**

0006 Understand the properties of linear functions and relations.

For example:

- analyzing a linear equation in terms of slope and intercepts
- determining the linear function that best models a set of data
- solving systems of linear equations and inequalities using a variety of techniques (e.g., algebraic, graphic, matrix)
- applying properties of linear equations and inequalities to model and solve a variety of real-world problems
- using techniques of linear programming to model and solve real-world problems
- determining connections among proportions, linear functions, and constant rates of change

0007 Understand the properties of quadratic and higher-order polynomial functions and relations.

For example:

- analyzing the characteristics of the roots of quadratic and higher-order polynomial functions
- solving systems of quadratic equations or inequalities using a variety of techniques (e.g., factoring, graphing, completing the square, quadratic formula)
- modeling and solving a variety of mathematical and real-world problems involving quadratic functions and relations
- analyzing symbolic, graphic, or tabular representations of a given quadratic or higher-order polynomial relation
- analyzing the results of changing parameters on the graphs of quadratic and higher-order polynomial functions
- applying properties of polynomials to model and solve problems

**FIELD 04: MATHEMATICS
TEST FRAMEWORK**

0008 Understand the properties of rational, radical, and absolute value functions and relations.

For example:

- analyzing the properties of a given function (e.g., range, domain, asymptotes)
- solving problems involving rational, radical, and absolute value functions using various algebraic techniques
- modeling and solving problems graphically using systems of equations and inequalities involving rational, radical, and absolute value relations (including the use of graphing calculators)
- interpreting and analyzing the effects of transformations on the graph of a function
- applying the properties of rational, radical, and absolute value functions and relations to model and solve problems

0009 Understand the properties of exponential and logarithmic functions.

For example:

- analyzing the relationship between logarithmic and exponential functions
- applying the laws of logarithms to manipulate and simplify expressions
- analyzing the graph of a logarithmic or exponential function or relation
- solving problems involving exponential growth and decay
- modeling and solving problems analytically or graphically using exponential and logarithmic relations

**FIELD 04: MATHEMATICS
TEST FRAMEWORK**

SUBAREA III—TRIGONOMETRY AND CALCULUS

0010 Understand principles, properties, and relationships involving trigonometric functions and their associated geometric representations.

For example:

- using degree and radian measures
- analyzing connections and identities among right triangle ratios, trigonometric functions, and the unit circle
- analyzing characteristics of the graph of a trigonometric function (e.g., frequency, period, amplitude, phase shift)
- determining connections between trigonometric functions and power series
- using the complex exponential function to explore properties of trigonometric functions

0011 Apply the principles and techniques of trigonometry to model and solve problems.

For example:

- applying trigonometric functions to solve problems involving length, area, volume, or angle measure (e.g., arcs, angles and sectors associated with a circle, unknown sides and angles of polygons, vectors)
- using circular functions to model periodic phenomena
- solving trigonometric equations using analytic or graphing techniques
- modeling and solving problems involving trigonometric functions

**FIELD 04: MATHEMATICS
TEST FRAMEWORK**

0012 Demonstrate an understanding of the fundamental concepts of calculus.

For example:

- analyzing the concept of limit numerically, algebraically, graphically, and in writing
- interpreting the derivative as the limit of the difference quotient
- interpreting the definite integral as the limit of a Riemann sum
- applying the fundamental theorem of calculus
- applying concepts of derivatives to interpret gradients, tangents, and slopes
- applying the concept of limit to analyze and interpret the properties of functions (e.g., continuity, asymptotes)
- applying the concept of rate of change to interpret statements from science, technology, economics, and other disciplines

0013 Apply the principles and techniques of calculus to model and solve problems.

For example:

- using derivatives to model and solve real-world problems (e.g., rates of change, related rates, optimization)
- applying properties of derivatives to analyze the graphs of functions
- using integration to model and solve problems (e.g., the area under a curve, work, applications of antiderivatives)
- modeling and solving problems involving first order differential equations (e.g., separation of variables, initial value problems)

**FIELD 04: MATHEMATICS
TEST FRAMEWORK**

SUBAREA IV—MEASUREMENT AND GEOMETRY

0014 Understand and apply measurement principles.

For example:

- applying appropriate measurement tools and units and converting within and between measurement systems (e.g., conversion factors, dimensional analysis)
- recording measurements to the appropriate degree of accuracy and analyzing the effect of uncertainty on derived measures
- deriving and applying formulas to find measures such as length, angle, area, and volume for a variety of geometric figures
- using nets and cross sections to determine volume and surface area formulas of three-dimensional figures
- applying the Pythagorean theorem to solve measurement problems

0015 Understand the principles and properties of axiomatic (synthetic) geometries.

For example:

- using the properties of lines and angles (e.g., parallelism, perpendicularity, supplementary angles, vertical angles) to characterize geometric relationships
- using the concepts of similarity and congruence to analyze the properties of geometric figures (e.g., triangle, parallelogram, polygon, circle)
- analyzing procedures used in geometric constructions (e.g., constructing the perpendicular bisector of a given line segment)
- proving theorems using an axiomatic system
- analyzing, comparing, and contrasting the axiomatic structures and properties of various geometries (e.g., Euclidean, non-Euclidean, projective)
- applying geometric principles to analyze three-dimensional figures

**FIELD 04: MATHEMATICS
TEST FRAMEWORK**

0016 Understand the principles and properties of coordinate geometry.

For example:

- applying the principles of distance, midpoint, slope, parallelism, and perpendicularity to characterize coordinate geometric relationships
- using coordinate geometry to prove theorems about geometric figures (e.g., triangle, parallelogram, circle, parabola, hyperbola)
- representing two- and three-dimensional geometric figures in various coordinate systems (e.g., Cartesian, polar)
- analyzing and applying transformations in the coordinate plane
- applying the distance formula to derive the equation of a conic section
- modeling and solving problems using conic sections

0017 Apply mathematical principles and techniques to model and solve problems involving vector and transformational geometries.

For example:

- modeling and solving problems involving vector addition and scalar multiplication (e.g., force)
- applying principles of geometry to model and solve problems involving the composition of transformations (e.g., translations, reflections, dilations, rotations)
- analyzing how transformational geometry and symmetry are used in art and architecture (e.g., tessellations, tilings, frieze patterns, fractals)
- using multiple representations of geometric transformations (e.g., coordinate, matrix, diagrams)
- proving theorems using vector and transformational methods

**FIELD 04: MATHEMATICS
TEST FRAMEWORK**

**SUBAREA V—DATA ANALYSIS, PROBABILITY, STATISTICS, AND DISCRETE
MATHEMATICS**

0018 Understand the principles, properties, and techniques related to sequence, series, summation, and counting strategies and their applications to problem solving.

For example:

- using first- or second-order finite differences to analyze sequences
- modeling and solving problems using the properties of sequences and series (e.g., arithmetic, geometric, Fibonacci)
- solving a variety of problems involving permutations and combinations
- analyzing the relationship between the binomial coefficients and Pascal's triangle

0019 Understand the principles, properties, and techniques of probability and their applications.

For example:

- evaluating the probability of events (e.g., joint, conditional, independent, mutually exclusive)
- interpreting graphic representations of probabilities (e.g., tables, charts, Venn diagrams, tree diagrams, frequency graphs, the normal curve)
- modeling and solving problems involving uncertainty using the techniques of probability (e.g., addition and multiplication rules, Bernoulli experiment)
- using simulations to estimate probabilities and analyzing the relationships between probability and statistics
- solving problems involving random variability and probability distributions (e.g., binomial, normal)

**FIELD 04: MATHEMATICS
TEST FRAMEWORK**

0020 Understand the principles, properties, and techniques of data analysis and statistics.

For example:

- applying measures of central tendency, dispersion, and skewness to summarize and interpret data presented in graphic, tabular, or pictorial form
- evaluating the statistical claims made for a given set of data (e.g., analyzing assumptions made in the sampling, analysis, and testing of statistical hypotheses) and how measures of reliability vary by discipline
- interpreting the outcomes of a given statistical test (e.g., *t*-test, chi-square analysis, correlation, linear regression)
- using graphing calculators to analyze and interpret data from a variety of disciplines (e.g., sciences, social sciences, technology)
- analyzing data using linear, logarithmic, exponential, and power regression models

0021 Understand how techniques of discrete mathematics (e.g., diagrams, graphs, matrices, propositional statements) are applied in the analysis, interpretation, communication, and solution of problems.

For example:

- representing finite data using a variety of techniques
- representing real-world situations and relationships using sequences and recurrence relations
- modeling and solving problems using graphs and matrices
- evaluating the use of computers and calculators to solve problems (e.g., developing and analyzing algorithms)

SUBAREA VI—ALGEBRA: CONSTRUCTED-RESPONSE ASSIGNMENT

The content to be addressed by the constructed-response assignment is described in Subarea II, Objectives 04–09.